The Ideal Gas Law

1. What is the Ideal Gas Law Equation?

$$\boxed{PV = nRT}$$

P = pressure (atm)
V = volume (L)
n = moles
R = universal gas constant $= 0.08206 \frac{\text{L atm}}{\text{mol K}}$
T = temperature (K)

K = Kelvin = 273.15 + °C
atm = atmospheres = 760 torr

2. From what laws is this equation derived?

   i. Boyle's Law – relationship between pressure and volume

a constant

$$PV = k$$

Notice that P and V are inversely proportional

   ii. Charles' Law – relationship between volume and temperature

a constant

$$V = bT$$

Notice that V and T are directly proportional

   iii. Avogadro's Law – relationship between moles and volume

$$V = \overset{\text{a constant}}{a}\,n$$

Notice that n and V are directly proportional

3. A sample of hydrogen gas has a volume of 8.56 L at a temperature of 0^{o}C and a pressure of 1.5 atm.

   a. Calculate the moles of hydrogen present in the sample.

   You will pretty much use the ideal gas law equation for 95% of the problems that you do – so make sure you are comfortable with it. It is really important to stay on top of your units when solving – these are pretty straightforward question – it is typically the details that will catch you.

   The best way to start is to organize the information you have been as it makes it much more straight forward to determine what to solve for and how to solve for it.

$$P = 1.5\ atm \qquad T = 0 + 273.15 = 273.15K$$

$$V = 8.56\ L \qquad n = ?$$

$$R = 0.08206\ \frac{L\ atm}{mol\ K}$$

   As you can see you are using all the variable in the ideal gas law… now all you need to do is plug in and solve.

Reorganizing the ideal gas law to solve for n

$$n = \frac{P\,V}{R\,T}$$

$$n = \frac{(1.5\ atm)\ (8.56\ L)}{\left(0.08206\ \frac{L\ atm}{mol\ K}\right)(273\ K)} = \boxed{0.57\ mol}$$

b. How would this answer change if the gas had been helium?

There would be no difference in the answer. There isn't anything identifying in the ideal gas law. It is solely based on the conditions of the gas. Why this is will become clearer when we get to the postulates of the kinetic molecular theory.

4. What equation can you use if you are given a question in which there a change in state?

$$\frac{P_1\,V_1}{n_1\,T_1} = R \qquad \frac{P_2\,V_2}{n_2\,T_2} = R$$

thus

$$\frac{P_1\,V_1}{n_1\,T_1} = \frac{P_2\,V_2}{n_2\,T_2}$$

Because both conditions set would be equal to the same constant, R, they are equivalent. This equation is really useful and easily simplified. If there is a condition that does not change it can be removed from the equation. For example, if you were given situation in which moles of the substance and temperature remained constant the equation would simplify to…

$$P_1\,V_1 = P_2\,V_2$$

5. A sealed balloon is filled with 1.00 L of helium at 23°C and 1.00 atm. The balloon rises to a point in the atmosphere where the pressure is 220. torr and the temperature is -31°C. What is the

change in the volume of the balloon as it ascends from 1.00 atm → 220. torr?

$P_1 = 1.00\ atm$ $\quad$ $P_2 = 220.\ torr \ \dfrac{1\ atm}{760\ torr} = 0.289\ atm$

$V_1 = 1.00L$ $\quad$ $V_2 = ?$

$T_1 = 23^oC + 273 = 296\ K$ $\quad$ $T_2 = -31^oC + 273 = 242K$

n can be ignored because the question stated "sealed balloon" meaning moles won't change.

Based on the information, your equation would be

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Now, simply plug in and solve

$$\frac{1.00\ atm\ \ 1.00L}{296\ K} = \frac{0.289\ atm\ \ V_2}{242K}$$

$$V_2 = 2.83\ L$$

Careful now… this is NOT the answer. The question asks for the <u>change</u> in volume. So you actually have one more step to complete…

$$2.83L - 1.00\ L = \boxed{1.83L}$$

6. A bicycle tire is filled with air to a pressure of 75 psi at a temperature of 19°C. Riding the bike on asphalt on a hot day increases the temperature of the tire to 58°C. The volume of the tire increases by 4.0%. What is the new pressure in the bicycle tire?

$P_1 = 75\ psi$ $\quad$ $P_2 = ?$

$V_1 = x$ $\quad$ $V_2 = x + 0.04x = 1.04x$

$T_1 = 19^oC + 273 = 292\ K$ $\quad$ $T_2 = 58^oC + 273 = 331K$

n can be ignored because moles won't change.

As in the previous example, based on the given information, solve with

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{(75\ psi)\ \cancel{(x)}}{(292K)} = \frac{(P_2)\ (1.04\cancel{x})}{331K} = \boxed{81.7\ psi}$$

7. Assuming constant temperature and pressure what would the final volume be for the following reaction after it has run to completion?

$$2\ NO_{2\,(g)} \rightarrow N_2O_{4\,(g)}$$
$$V_i = 38.6\ mL \qquad V_f = ?$$

The temperature and pressure remain constant. So only the change in moles will affect the volume. We were not given the moles… but we can use the stoichiometric coefficient. There is no worry about a limiting reactant in this particular problem as it relies on one reactant. Additionally the reaction is running to completion – we will therefore have half as many moles as we started out with.

$$n_1 = 2\ mol \qquad n_2 = 1\ mol$$
$$V_1 = 38.6\ mL \qquad V_2 = ?$$

We can take this information and plug it into the change of state equation.

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

Remember in the ideal gas law there is no distinction between molecules – so there is no issue due to the fact that in this reaction we are changing from NO_2 to N_2O_4.

$$\frac{38.6\ mL}{2\ mol} = \frac{?}{1\ mol}$$

$$V_2 = \boxed{19.3\ mL}$$

8. What is STP?

An acronym that stands for **S**tandard **T**emperature and **P**ressure.

Standard Temperature = 273K
Standard Pressure = 1 atm

9. Consider

$$2NaN_{3\,(s)} \rightarrow 2Na_{\,(s)} + 3N_{2\,(g)}$$

What mass of NaN_3 must be reacted in order to inflate an airbag to 71.4 L at STP?

This is a fantastic example of a good test question. The problem itself is not very difficult – it does require you to understand the information you are given. There is a pitfall to this question that many students tend to fall for.

Let's look at the information given to us…

$V = 71.4\ L$
$P = 1\ atm$
$T = 273\ K$

Looking at this data and what the question is asking us for (the mass of sodium azide, $NaN_{3.}$), what would make the most sense to solve for? Hopefully you came to the conclusion that n, the moles, makes the most sense. We are in the gas chapter and we have 3 out of the 4 variable from the ideal gas law. Additionally, it is not a far leap to go from moles to mass… so it would get us much closer to obtaining the answer to the question.

$$n = \frac{(1\ atm)\ (71.4\ L)}{\left(0.08206\ \frac{L\ atm}{mol\ K}\right)\ (273K)} = 3.19\ mol$$

This is the sticky part – identifying which moles you just solved for. Does this number pertain to NaN_3, Na or N_2? Many students assume NaN_3 because that is the substance that question focuses on – but this is where paying attention to the detail comes in. Why could it not possibly be NaN_3? Think about the equation you used to solve – can you use the ideal GAS law to solve directly for the moles of a solid? No. This means that we could have only solved for the moles of N_2 as it is the only gas in the reaction equation.

So now we have to use stoichiometry to finish up the problem.

$$3.19\ mol\ N_2 \quad \frac{2\ mol\ NaN_3}{3\ mol\ N_2} \quad \frac{65.02\ g\ NaN_3}{1\ mol\ NaN_3} = \boxed{138\ g\ NaN_3}$$

10. A compound has the empirical formula CHCl. A 256 mL flask, at 373K and 750 torr contains 0.800g of the gaseous compound. Give the molecular formula.

There are a couple things you need to remember to complete this question.
First, the relationship between the empirical formula and molecular formula.

$$\frac{\text{Molar Mass of the Molecular Formula}}{\text{Molar Mass of the Empirical Formula}} = \text{Factor}$$

Once you have determined the factor – you simply multiply the subscripts in formula by the factor to obtain the molecular formula.

Getting the empirical formula mass is not an issue. The question becomes how to obtain the molar mass of the molecular formula. There are two ways to go about doing this.

Method 1

$$Molar\ Mass = \frac{grams}{moles}$$

Looking at the information provided in the question

$$mass = 0.800\ g$$
$$P = \left(750\ torr \frac{1\ atm}{760\ torr}\right) = 0.987\ atm$$
$$V = 0.256\ L$$
$$T = 373\ K$$
$$n = ?$$

The only item we need to solve for is moles. We can do this by using PV=nRT as we have all the variables necessary to solve for n.

$$n = \frac{(0.987\ atm)\ (0.256\ L)}{\left(0.08206 \frac{L\ atm}{mol\ K}\right)(373\ K)} = 8.25 \times 10^{-3}\ mol$$

$$Molar\ Mass = \frac{0.800g}{8.25 \times 10^{-3}\ mol} = 96.9\ g/mol$$

Method 2

Use the formula

$$Molar\ Mass = \frac{dRT}{P}$$
$$\text{where } d = \text{density } (g/L)$$

$$d = \frac{0.800g}{0.256\ L} = 3.13 \frac{g}{L}$$

$$Molar\ Mass = \frac{(3.13\frac{g}{L})\left(0.08206\ \frac{L\ atm}{mol\ K}\right)(373\ K)}{(0.987\ atm)} = 96.9\ g/mol$$

Now plug into formula to obtain factor:

$$\frac{96.9\ g/mol}{48.47\ g/mol} = 2$$

Molecular Formula

$$\boxed{C_2H_2Cl_2}$$

11. Consider:

$$\underset{3.0\ L}{4NH_{3\ (g)}} + \underset{6.0L}{5O_{2\ (g)}} \rightarrow 4NO_{\ (g)} + 6H_2O_{\ (g)}$$

If this reaction is run at STP, what would the total volume be after this reaction has run to completion.

All gases at STP – therefore pressure and temperature remained constant.

Thus we can use this equation

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

Remember that it doesn't matter that there was a reaction – under the ideal gas law – all gases are fundamentally the same. So use the volumes provided and use the stoichiometric coefficients for the values of n.

This is a problem where they provided information about both reactants – these means you must figure out which is the limiting reactant.

$$\frac{3.0\ L\ NH_3}{4\ mol\ NH_3} = \frac{x\ L\ NO}{4\ mol\ NO} = 3.0\ L\ NO \quad (limiting\ reactant)$$

$$\frac{6.0\ L\ O_2}{5\ mol\ O_2} = \frac{x\ L\ NO}{4\ mol\ NO} = 4.8\ L\ NO$$

Because NH_3 is the limiting reactant… we have to use it to determine the total amount of product (both H_2O and NO – as they are both gaseous) formed.

$$\frac{3.0\ L\ NH_3}{4\ mol\ NH_3} = \frac{x\ L\ H_2O}{6\ mol\ H_2O} = 4.5\ L\ H_2O$$

Last portion to remember is that the total volume consists of more than just the product – it also contains the excess gaseous O_2 that did not react.

$$\frac{3.0\ L\ NH_3}{4\ mol\ NH_3} = \frac{x\ L\ O_2\ needed}{5\ mol\ O_2} = 3.75\ L\ of\ O_2\ needed$$

$$6.0L\ of\ O_2\ initially\ -\ 3.75\ L\ of\ O_2\ required = 2.25L\ of\ O_2\ remaining$$

$$3.0\ L + 4.5\ L + 2.25\ L = \boxed{9.75\ L}$$